

Composite vertical post near window load bearing in long term-duration loading

Wood type:	C24
Width of wooden element:	$b := 45 \text{ mm}$
Height of wooden element:	$h := 95 \text{ mm}$
Thickness of plywood element:	$b_w := 21 \text{ mm}$
Width of plywood element:	$H := 400 \text{ mm}$
Clear width between two wood elements:	$h_w := 210 \text{ mm}$
Plywood between two posts area:	$A_w := b_w \cdot H = 84 \text{ cm}^2$
For I-beam section [EC5 9.1.1 (8)]:	$b_{ef} := \frac{b_w}{2} = 10.5 \text{ mm}$
Buckling length about x axis (minus minimal lintel height):	$l_x := 2.886 \text{ m} - 0.4 \text{ m} = 2.486 \text{ m}$
Buckling length about y axis:	$l_y := 1.007 \text{ m}$
Factor for support condition at the ends of the element:	$\mu_x := 1 \quad \mu_y := 1$
Factor for solid timber straightness:	$\beta_c := 0.2$
Charac. wood bending strength:	$f_{m,k} := 24 \text{ MPa}$
Charac. wood compression strength parallel to the grain:	$f_{c,0,k} := 21 \text{ MPa}$
Charac. wood tension strength parallel to the grain:	$f_{t,0,k} := 14 \text{ MPa}$
Wood modulus of elasticity parallel to the grain:	$E_{0,05} := 7400 \text{ MPa}$
Wood modulus of mean elasticity parallel to the grain:	$E_{0,mean} := 11000 \text{ MPa}$
Charac. plywood compression strength:	$f_{p,c,90,k} := 25 \text{ MPa}$
Charac. plywood tensile strength :	$f_{p,t,90,k} := 36 \text{ MPa}$
Charac. plywood panel shear strength :	$f_{p,v,k} := 9.5 \text{ MPa}$
Charac. plywood rolling shear strength :	$f_{p,r,k} := 2.59 \text{ MPa}$
Plywood modulus of mean elasticity:	$E_{p,c,90,mean} := 7642 \text{ MPa}$
Plywood mean modulus of rigidity:	$G_{w,mean} := 620 \text{ MPa}$

Factor for duration loading and service:

$$k_{mod} := 0.7$$

Factor for load shearing:

$$k_{sys} := 1.0$$

Factor for solid timber - bending and axial tension:

$$k_h := \min \left(\left(\frac{150 \text{ mm}}{h} \right)^{0.2}, 1.3 \right) = 1.096$$

Deformation factor for for solid timber class 1:

$$k_{def,f} := 0.6$$

Deformation factor for for plywood class 1:

$$k_{def,w} := 0.8$$

Partial factor for wood properties:

$$\gamma_{M,f} := 1.30$$

Partial factor for plywood properties:

$$\gamma_{M,w} := 1.25$$

Buckling resistance condition with beam
laterally supported along its compression
flange:

$$k_c := 1$$

Geometric properties - transformed sections

Transformed web thickness (into wood):

$$b_{w.tfd} := \left(b_w \cdot \frac{E_{p.c.90.mean}}{E_{0.mean}} \right) = 14.589 \text{ mm}$$

Second moment of area of the flanges (into wood):

$$I_{eff,x} := \frac{b \cdot H^3}{12} - \frac{b \cdot (H - 2 \cdot h)^3}{12} = (2.053 \cdot 10^8) \text{ mm}^4$$

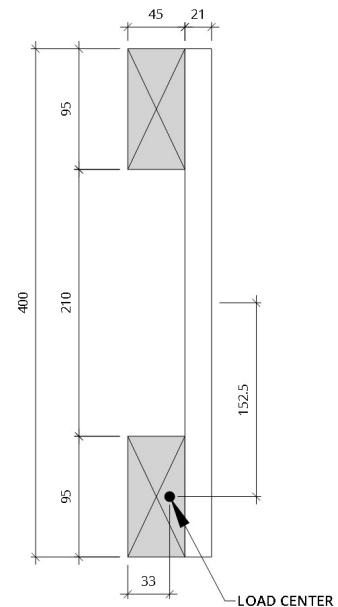
$$I_{eff,y} := \frac{b^3 \cdot H}{12} - \frac{b^3 \cdot (H - 2 \cdot h)}{12} = (1.443 \cdot 10^6) \text{ mm}^4$$

Second moment of area of the web:

$$I_{ef,w,x} := \frac{b_{w.tfd} \cdot H^3}{12} = (7.781 \cdot 10^7) \text{ mm}^4 \quad I_{ef,w,y} := \frac{b_{w.tfd}^3 \cdot h}{12} = (2.458 \cdot 10^4) \text{ mm}^4$$

Instantaneous second moment of the tranformed section:

$$I_{ef,x} := I_{eff,x} + I_{ef,w,x} = (2.831 \cdot 10^8) \text{ mm}^4 \quad I_{ef,y} := I_{eff,y} + I_{ef,w,y} = (1.467 \cdot 10^6) \text{ mm}^4$$



Pic1. Analyzed cross-section

Factor for quasi-permanent value of variable action:

$$\psi_2 := 0.3$$

Transformed web thickness (into wood):

$$b_{c.w.tfd} := \left(b_w \cdot \frac{E_{p.c.90,mean}}{E_{0,mean}} \right) \cdot \frac{1 + \psi_2 \cdot k_{def,f}}{1 + \psi_2 \cdot k_{def,w}} = 13.883 \text{ mm}$$

$$I_{c.ef,w,x} := \frac{b_{c.w.tfd} \cdot H^3}{12} = (7.404 \cdot 10^7) \text{ mm}^4 \quad I_{c.ef,w,y} := \frac{b_{c.w.tfd}^3 \cdot H}{12} = (8.92 \cdot 10^4) \text{ mm}^4$$

$$I_{c.ef,x} := I_{ef,f,x} + I_{c.ef,w,x} = (2.793 \cdot 10^8) \text{ mm}^4 \quad I_{c.ef,y} := I_{ef,f,y} + I_{c.ef,w,y} = (1.532 \cdot 10^6) \text{ mm}^4$$

Cross-sectional area:

$$A := b \cdot h \cdot 2 + A_w = 169.5 \text{ cm}^2$$

Second moment of area:

$$I_x := I_{c.ef,x} \quad I_x = (2.793 \cdot 10^8) \text{ mm}^4 \quad I_y := I_{c.ef,y} \quad I_y = (1.532 \cdot 10^6) \text{ mm}^4$$

$$W_x := \frac{b_{c.w.tfd} \cdot H^2 + 2 \cdot b \cdot H^2 - 2 \cdot b \cdot (H - 2 \cdot h)^2}{6} = (2.109 \cdot 10^6) \text{ mm}^3$$

$$W_y := \frac{b_{c.w.tfd}^2 \cdot H + (2 \cdot b)^2 \cdot H - (2 \cdot b)^2 \cdot (H - 2 \cdot h)}{6} = (2.693 \cdot 10^5) \text{ mm}^3$$

Radius of gyration:

$$i_x := \sqrt{\frac{I_x}{A}} = 128.37 \text{ mm} \quad i_y := \sqrt{\frac{I_y}{A}} = 9.507 \text{ mm} \quad i := \min(i_x, i_y) = 9.507 \text{ mm}$$

Design element length:

$$l_{ef,x} := \mu_x \cdot l_x = 2.486 \text{ m} \quad l_{ef,y} := \mu_y \cdot l_y = 1.007 \text{ m}$$

Slenderness ratio:

$$\lambda_x := \frac{l_{ef,x}}{i_x} = 19.366 \quad \lambda_y := \frac{l_{ef,y}}{i_y} = 105.921 \quad \lambda := \max(\lambda_x, \lambda_y) = 105.921$$

Relative slenderness:

$$\lambda_{rel,x} := \frac{\lambda_x}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = 0.328$$

$$\lambda_{rel,y} := \frac{\lambda_y}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = 1.796$$

Instability factor:

$$k_x := 0.5 \cdot (1 + \beta_c \cdot (\lambda_{rel,x} - 0.3) + \lambda_{rel,x}^2) = 0.557$$

$$k_y := 0.5 \cdot (1 + \beta_c \cdot (\lambda_{rel,y} - 0.3) + \lambda_{rel,y}^2) = 2.263$$

$$k_{c,x} := \frac{1}{k_x + \sqrt{k_x^2 - \lambda_{rel,x}^2}} = 0.994$$

$$k_{c,y} := \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = 0.275$$

Axial strength in compression:

$$f_{c,0,d} := \frac{k_{mod} \cdot k_{sys} \cdot f_{c,0,k}}{\gamma_{M,f}} = 11.308 \text{ MPa}$$

Axial strength in tension:

$$f_{t,0,d} := \frac{k_{mod} \cdot k_h \cdot k_{sys} \cdot f_{t,0,k}}{\gamma_{M,f}} = 8.26 \text{ MPa}$$

Compressive stress/Design buckling strength equation :

$$N_{c,k,x} := f_{c,0,k} \cdot A \cdot k_{c,x} = 353.701 \text{ kN}$$

$$N_{c,k,y} := f_{c,0,k} \cdot A \cdot k_{c,y} = 97.828 \text{ kN}$$

$$N_{c,d,x} := f_{c,0,d} \cdot A \cdot k_{c,x} = 190.454 \text{ kN}$$

$$N_{c,d,y} := f_{c,0,d} \cdot A \cdot k_{c,y} = 52.676 \text{ kN}$$

$$\sigma_{c,0,d,x} := \frac{N_{c,d,x}}{A} = 11.236 \text{ MPa}$$

$$\sigma_{c,0,d,y} := \frac{N_{c,d,y}}{A} = 3.108 \text{ MPa}$$

Design wood bending strength:

$$f_{m,d} := \frac{k_{mod} \cdot k_h \cdot k_{sys} \cdot f_{m,k}}{\gamma_{M,f}} = 14.159 \text{ MPa}$$

Moment about the axis x-x:

$$e_y := \frac{H}{2} - \frac{h}{2} = 152.5 \text{ mm}$$

$$e_x := \frac{b_w}{2} + \frac{b}{2} = 33 \text{ mm}$$

$$M_{x,d} := N_{c,d,x} \cdot e_y = 29.044 \text{ kN} \cdot \text{m}$$

$$M_{y,d} := N_{c,d,y} \cdot e_x = 1.738 \text{ kN} \cdot \text{m}$$

Bending strength of the web in compression:

$$f_{c,w,d} := \frac{k_{mod} \cdot k_{sys} \cdot f_{p,c,90,k}}{\gamma_{M,w}} = 14 \text{ MPa}$$

Bending stress in top and the bottom flange:

$$\sigma_{f.c.max.d.x} := \frac{M_{x.d}}{I_{c.ef.x}} \cdot \frac{H}{2} = 20.797 \text{ MPa}$$

$$\sigma_{f.c.max.d.y} := \frac{M_{y.d}}{W_y} = 6.454 \text{ MPa}$$

Bending strength of the web in tension:

$$f_{c.w.d} := \frac{k_{mod} \cdot k_{sys} \cdot f_{p.t.90.k}}{\gamma_{M.w}} = 20.16 \text{ MPa}$$

Bending stress in the web:

$$\sigma_{w.c.max.d.x} := \frac{M_{x.d}}{I_{ef.x}} \cdot \frac{H}{2} \cdot \left(\frac{E_{p.c.90.mean}}{E_{0.mean}} \right) = 14.256 \text{ MPa}$$

$$\sigma_{w.c.max.d.y} := \frac{M_{y.d}}{W_y} \cdot \left(\frac{E_{p.c.90.mean}}{E_{0.mean}} \right) = 4.484 \text{ MPa}$$

Axial stress in the top and bottom flange:

$$\sigma_{f.c.max.d.c} := \frac{M_{x.d}}{I_{c.ef.x}} \cdot e_y = 15.858 \text{ MPa}$$

Design buckling strength:

$$k_c \cdot f_{c.0.d} = 11.308 \frac{\text{N}}{\text{mm}^2}$$

Checking combined compression and bending stress condition about the x-x axis:

$$a := \frac{\sigma_{c.0.d.x}}{k_{c.x} \cdot f_{c.0.d}} + \frac{\sigma_{f.c.max.d.x}}{f_{m.d}} = 2.469$$

$$N_{c.d.x,final} := \frac{N_{c.d.x}}{a} = 77.145 \text{ kN}$$

$$b := \frac{\sigma_{c.0.d.y}}{k_{c.y} \cdot f_{c.0.d}} + \frac{\sigma_{f.c.max.d.y}}{f_{m.d}} = 1.456$$

$$N_{c.d.y,final} := \frac{N_{c.d.y}}{b} = 36.184 \text{ kN}$$

$$N_{c.d,max} := N_{c.d,y,final} = 36.184 \text{ kN}$$

